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THE DEVELOPMENT AND SUBSTANTIATION OF THE KINETIC THEORY OF GASES IN THE TWENTIETH CENTURY†

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A new kinetic theory that is close to dynamic processes is constructed. A system of M integro-differential equations and a system of M partial differential equations are obtained. The theory is demonstrated using the examples of the calculation of the structure of intense shock-waves and by calculating turbulent flows in a plane channel. It is shown that the theory of the structure of high-intensity shock-waves agrees with remarkable accuracy with numerous experimental data. Calculations of turbulent flow approximate quite well to experimental data, but it is remarkable that a single theory can describe both the turbulent core at the centre of the channel and the laminar sublayer on the wall so well. © 1997 Elsevier Science Ltd. All rights reserved.

The kinetic theory of gases was founded at the end of the last century by Boltzmann. Using the laws of classical mechanics and qualitative intuitive ideas he obtained the famous integro-differential kinetic equation. The first approximate solutions were given by Boltzmann himself. He verified the hydrodynamic equations taking viscous friction into account.

The basic ideas of Boltzmann's kinetic theory were subject to serious criticism by scientists (his contemporaries), which he could not convincingly refute. Loschmidt first pointed out the symmetry of the laws of mechanics with respect to the past and the future; this symmetry does not exist in Boltzmann's kinetic theory.

To obtain the kinetic equation Boltzmann assumed the hypothesis of molecular chaos, which still gives rise to doubts related to the following facts.

In strictly elastic collisions between molecules all the conservation laws are satisfied, and at the molecular level an ordered distribution of correlated velocities is formed. Hence, the integrands in Boltzmann's theory are themselves invariant under elastic collisions; we cannot include the idea of chaos here.

Zermelo drew attention to Poincaré's theorem, according to which a system of material particles has a quasi-periodic form of motion. Boltzmann had carried out the proper calculations for a set of particles in 1 cm^3 under normal conditions. For reasonable requirements on the accuracy of recovery, this is justified, but the required recovery time will be enormous. The quasi-periodicity is proved. Hence, recovery is only justified when there is no chaos.

In our time the leading scientists of the world N. N. Bogolyubov *et al.*, using rigorous mathematical methods, have derived the Boltzmann equation from Liouville's equation without invoking the hypothesis of molecular chaos.

Boltzmann also showed that the entropy of matter in a closed volume increases, while its ability to do work decreases—matter tends to chaos and to thermal death. Boltzmann also put forward the so-called fluctuation hypothesis, which introduced some calm, but could not entirely satisfy his opponents.

The fundamental propositions of Boltzmann's theory have received fundamental theoretical developments and justifications in the present century.

1. At the beginning of the century several different methods of solving the Boltzmann equation were put forward by Hilbert [1], Chapman–Enskog [2, 3] and also by Grad [4] and others with numerous modifications. These methods led to different expressions when solving the same problems, but, as a rule, the results obtained were identical and physically clear.

2. In the middle of the present century there was increased interest in the kinetic theory of gases throughout the world, due to the rapid development of high-speed aviation and space techniques. In 1946 Bogolyubov [5], Born and Green [6] and Kirkwood [7], practically simultaneously, published fundamental results in the kinetic theory.

Starting from Liouville's equation and the various kinetic equations that follow from it, these scientists succeeded in obtaining the Boltzmann kinetic equation strictly mathematically.

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In [8, 9] we also derived the Boltzmann kinetic equation from Liouville's equation taking into account a singular integral form of the interaction, not inherent in classical mechanics, since the molecules behave here as non-localized particles. By considering this integral form of interaction it was possible to construct a rigorous irreversible solution of the chain of kinetic equations and to derive the Boltzmann kinetic equation. A consideration of this integral form of the interaction in fact indicates that the distribution function of the probability of finding particles in a six-dimensional phase space depends differently on fast and slow time.

In addition, we showed that Hilbert's method does not enable a correct solution of the Boltzmann kinetic equation to be constructed [10, 11], and a new more general method of solution was proposed [12] compared with those obtained in the publications mentioned above. In this method the fact that the distribution function depends both on fast and slow time is taken into account. This method is in fact based on the ideas of Poincaré's method [13].

It has become clear from the above theoretical investigations that during the last century the distinguished scientist Boltzmann succeeded in obtaining the kinetic equation for gases and the equations of hydrodynamics which follow from it, which were given a rigorous theoretical basis in the twentieth century.

3. Moreover, Boltzmann obtained kinetic equations for gaseous mixtures describing the motion of the individual components of the gas. For an M-component gaseous mixture this kinetic equation was written in the form

$$\frac{df_s}{dt} = \frac{1}{\varepsilon}\sum_{\tau=1}^{M}\iiint \left(f_\tau' f_s' - f_\tau f_s\right) q_{\tau s} b\,db\,d\varepsilon\,d\mathbf{v}_\tau = \sum_{\tau=1}^{M} I(f_\tau f_s) \tag{1}$$

where $f_s(t, r, \mathbf{v}) = f_s$ is the probability distribution function of the sth component of the mixture in six-dimensional phase space $q_{\tau s}$ is the initial relative velocity of these molecules, b is their impact parameter, $1/\varepsilon$ is a measure of the collision frequency, and ε is a small quantity. The distribution function can be expanded in the parameter

$$f_s(t, \mathbf{r}, \mathbf{v}) = f_s^{(0)}(t, \mathbf{r}, \mathbf{v}) + \varepsilon f_s^{(1)}(t, \mathbf{r}, \mathbf{v}) + \varepsilon^2 f_s^{(2)}(t, \mathbf{r}, \mathbf{v}) + \ldots \tag{2}$$

In the first half of the present century the same Chapman–Enskog method [2] was used to solve the system of Boltzmann kinetic equations.

To solve the system of kinetic equations using this method for gaseous mixtures in the zeroth approximation it was found to be necessary to require that the sum of all the collision integrals in the zeroth approximation should be zero, which follows from (1) and (2)

$$\sum_{\tau=1}^{M} I\left(f_\tau^{(0)}, f_s^{(0)}\right) = 0$$

This meant that the kinetic theory could only describe the motion of the gaseous mixture as a whole and could only determine the mean-mass parameters of the flow, while the motion of individual components in the Euler approximation remain completely undetermined, which also follows from well-known results.

4. Only at the beginning of the 1970s in Akademgorodok, Novosibirsk, was the problem of separating gaseous mixtures keenly pursued, but no theory existed, and we were therefore forced to develop a new method of solving the kinetic equation for a gaseous mixture. To describe the motion of a given component of a gaseous mixture successfully it must have its own integral equation with its own inherent integral kernel, whereas for the other components the integral kernels must be taken into account in the subsequent approximation.

The integral equation for a gaseous mixture was therefore written in the form

$$\varepsilon\left(\frac{df_s}{dt} - \sum_{\tau\neq s}^{M} I(f_\tau f_s)\right) = I(f_s f_s) \tag{3}$$

In this case it turned out to be possible to determine the partial parameters for each component of the flow, while the zeroth approximation was also written only for the main component of the mixture

$$I\left(f_\tau^{(0)}, f_s^{(0)}\right) = 0 \tag{4}$$

This method was presented in 1971 at seminars at the Moscow Institutes of the Academy of Sciences of the USSR and at Moscow University. In 1972 it was reported at the Eighth International Congress on Theoretical and Applied Mechanics in Moscow [14], and it was published in 1974 [15]. The method enables the motion and separation of multicomponent gaseous mixtures in tubes, channels and diffusors to be described, and also enables the motion of a dust-like catalyst in chemical engineering apparatus to be determined theoretically. Calculations were carried out in [16], while a modification of the Chapman–Enskog method for a two-temperature binary gaseous mixture was presented in [17].

The method of solving the system of Boltzmann equations for gaseous mixtures which we proposed enables the mean partial parameters of the flow (the density, velocity and temperature) to be determined for each component of the gaseous mixture

$$\rho_1, \rho_2, \ldots, \rho_M; \quad v_1, v_2, \ldots, v_M; \quad T_1, T_2, \ldots, T_M$$

From these partial parameters of the flow one can obtain, by simple summation, the mean-mass parameters, i.e. one can obtain the Chapman–Enskog solution.

In the publications mentioned above a multiparametric system of gas-dynamic equations

$$\frac{\partial v_s^\alpha}{\partial t} + v_s^\beta \frac{\partial v_s^\alpha}{\partial r_\beta} + \frac{1}{\rho_s}\frac{\partial p_s}{\partial r_\alpha} = \frac{16}{3\rho_s}\sum_{\tau \neq s}^{M} \frac{\rho_s \rho_\tau \Omega_{\tau s}^{(11)}}{m_\tau + m_s}\left(v_\tau^\alpha - v_s^\alpha\right) \tag{5}$$

$$\frac{\partial T_s}{\partial t} + v_s^\alpha \frac{\partial T_s}{\partial r_\alpha} + \frac{2}{3}T_s \frac{\partial v_s^\alpha}{\partial r_\alpha} = \frac{16}{3}\sum_{\tau \neq s}^{M} \frac{\rho_s \rho_\tau \Omega_{\tau s}^{(11)}}{m_\tau + m_s}\left\{\frac{3k}{2m}\left(T_\tau - T_s\right) + q_{\tau s}\left(\mathrm{v}_\tau - \mathrm{v}_s\right)^2\right\} \tag{6}$$

$$\frac{\partial \rho_s}{\partial t} + \frac{\partial \rho_s v_s^\alpha}{\partial r_\alpha} = 0 \tag{7}$$

was obtained for the first time, where $q_{\tau s} = (m_s T_s)/(T_\tau + T_s)$ and $\Omega_{\tau s}^{(11)}$ are tabulated integrals. Here an important form of interaction between high-speed flows $v_\tau^\alpha - v_s^\alpha$ was revealed for the first time, which plays a decisive role in science.

Investigations carried out in the twentieth century showed that the overwhelming majority of aerodynamic flows can be described completely satisfactorily by the Euler and Navier–Stokes equations, and also by the equations of multicomponent gas dynamics [15].

5. In the last century Boltzmann was so occupied with the law of the change of entropy of systems that we must touch on this problem at least briefly.

Starting from his kinetic equation, he showed that the entropy in closed volumes increases $dS/dt \geqslant 0$ and that the universe is approaching a thermal death. This proposition gave rise to some doubts and objections. To meet these objections Boltzmann put forward, as we have already noted, the so-called "fluctuation hypothesis", i.e. entropy fluctuates with time. A positive fluctuation $dS/dt > 0$ is followed by a negative fluctuation $dS/dt < 0$.

However, on the Earth, in our solar system, and in our galaxy, entropy cannot decrease, and dissipative processes take place intensively here.

At the same time, in the universe clusters of matter have been formed in which energy is stored but entropy decreases. The possibility that such formations can occur in the universe was predicted by Laplace. Rigorous theoretical results were obtained by Schwarzschild in 1916 on the basis of exact solutions of Einstein's equations for symmetrical cases. It was not until 1963 that Kerr succeeded in finding a solution for the Schwarzschild problem for the gravitation field of a rotating "black hole". Hence, it has been shown that material regions can exist in the universe with a different power source and different laws of variation of entropy.

For a specified region of outer space we will have $dS/dt \gtrless 0$.

Then, in our solar system and in our galaxy for a region without "black holes" $dS/dt > 0$, which follows from the given multiparametric kinetic theory, whereas in the Boltzmann theory $dS/dt \geqslant 0$.

6. During the twentieth century the methods of kinetic theory of matter were used to solve the overwhelming majority of aerodynamic problems and were highly effective. At the same time, a number of important aerodynamic problems were not understood and were not solved using the kinetic theory and the Navier–Stokes equations.

This applies to the following problems.

The structure of high-intensity shock-waves. Prandtl worked on this problem at the beginning of this century and was the first to encounter insurmountable difficulties. Calculations based on the Navier–

Stokes equations differed considerably from experimental data. Similar results were obtained by Mott–Smith and others.

Turbulent flows of a liquid and a gas. Reynolds, towards the end of the last century, on the basis of the simplest experiments using coloured liquid jets, concluded that turbulence in a flow is produced as a result of a loss of stability of the initial laminar flow. Using this idea Reynolds [18] distinguished all the flow parameters in the mean and under pulsation conditions. Throughout the twentieth century the problem of turbulence has occupied the leading scientists of the world: Prandtl, Karman, Heisenberg, Landau, and others.

Knudsen layers. Considerable difficulties arise when investigating thin Knudsen layers, which have several chaotic zones, which are not intermixed with one another. The thickness of these layers is less than the mean free path of the molecules.

Leading scientists of the world have expressed numerous points of view and specific proposals on these problems. The overwhelming majority of these have turned out to be largely ineffective.

In the 1950s in the U.S.A., the director of NASA constructed the first turbulence-free tube. Later, the director of the Institute of Mechanics of the Siberian Branch of the Academy of Sciences in Novosibirsk constructed the first turbulence-free tubes in the U.S.S.R. and conducted the first investigations. Tests on a flat plate in turbulence-free tubes led to a sharp increase in the critical Reynolds number to 3×10^6, whereas the stability limit in practice remained as before, namely, 6×10^4. There was an enormous transition region. No pulsations were observed in the front half of this region. In the rear half oscillations of considerable amplitude were observed, which increased as the transition point was approached. The complex pulsations that appeared in the transition zone indicate that spatial phenomena may serve as a basis for a new treatment of the transition problem.

It can be seen from the above material that the simple separation of the gas motion into small oscillations of an unstable nature and certain average (Reynolds) motions cannot be regarded as justified. In each specific case an exact treatment of the turbulence problem must exist on its own, but so far no such formulations have yet been obtained.

At the same time, there is a large class of various turbulent flows which are of considerable importance for mankind. In the middle of the present century these flows were investigated experimentally by Nikuradze [19] and by others. In particular, he showed that in circular tubes the velocity profile does not change along a length of many tens of bores. This most important result may, in particular, facilitate the solution of the turbulence problem. We will also use it later. But we must not oversimplify it. The complex perturbations that occur in the transition zone are certainly sustained along the whole length of the channel.

Problems of the structure of shock-waves and turbulent flows have not been solved within the framework of classical theory, which clearly does not embrace dynamics.

The kinetic theory is closer to thermodynamic processes. Even Boltzmann had $dS/dt \geqslant 0$. Hence, striking dynamic processes (shock-waves, turbulence, etc.) were difficult to explain by classical kinetic theory.

It follows from the above that to make any progress in the area of unsolved problems in mechanics we must: (1) make kinetic theory approximate to dynamics, and (2) we must direct the theory of these processes towards the use of a set of distribution functions.

To solve these problems we have used the possibility of employing Liouville's equation, which describes the dynamic property of systems when they are combined with a large thermostat. Liouville's equation will then be capable of describing the widest range of dynamic, gas-dynamic and statistical processes.

If at the initial instant the distribution function of the probabilities of particles residing in six-dimensional phase space is specified, Liouville's equation will be exactly equivalent mathematically to the initial Hamilton equations.

Gibbs also used a combination of a dynamic system with a thermostat and thereby obtained a micro-canonical distribution.

If a dynamic system resides for a certain time in a large thermostat with a specified property, distributions corresponding to it will be obtained in the dynamic system.

We draw attention to the fact that by proceeding from Liouville's equation and using corresponding thermostats, we are able to describe the following dynamic, kinetic and statistical systems: a dynamic Hamilton system, the multiparametric system of the author, Boltzmann's kinetic system, and Gibbs's statistical system.

The two penultimate cases are limiting states of the kinetic theory, when the most stable, two-particle and single-particle distribution functions are preserved. All the remaining distribution functions have already decayed.

The dynamic Hamilton system. We will first consider a purely dynamic system consisting of N different particles. We will write their probability distribution functions in six-dimensional phase space in the form

$$F_N = F_N(t, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) \tag{8}$$

These probability distribution functions will satisfy Liouville's equation and will differ from one another solely in the initial data. They will be asymmetrical functions with respect to the permutation of any pair of dynamic coordinates (a Hamilton system).

Gibbs's statistical system. We will now consider another limiting case: an ensemble consisting of N identical particles, the distribution functions of which also satisfy Liouville's equation and differ from one another solely in the initial data. If these ensembles have interacted for a long time with a certain thermostat, they will differ only slightly from one another and will be described by a symmetrical distribution function

$$F_N = F_N(0, t, \mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_N) \tag{9}$$

On the molecular level in the whole system, as Gibbs showed, the distribution function will be symmetrical with respect to a permutation of the dynamic coordinates, it does not depend on time, and can in this case be represented in the form

$$F_N = F_N^{\circ} \exp(-E/(kT)) \tag{10}$$

where E is the system energy, and the change in entropy is zero.

The Boltzmann kinetic system. We will now consider a set of ensembles of N identical particles which are described by a set of asymmetrical distribution functions (8), which also satisfy Liouville's equation and differ from one another solely in the initial data. If this ensemble of N particles has interacted elastically with a certain thermostat and the particles can mix strongly with one another, the mean flow parameters (the density, velocity and temperature) will be preserved in the system at the macroscopic level. At the molecular level an ordered distribution of the molecular velocities will be established in the whole system. The distribution function will be symmetrical

$$F_N = \prod_{k=1}^{N} \varphi_k(\mathbf{x} - \mathbf{a}) \tag{11}$$

The system will have exactly N single-particle distribution functions $f_s = (N/V)\varphi_1(x)$, where V is the volume of the system.

This case was considered by Boltzmann. The single-particle functions satisfy the kinetic equation

$$\frac{df}{dt} = \iiint \left(f' f_1' - f f_1\right) q b db d\varepsilon d\mathbf{v}_1 \tag{12}$$

obtained by Boltzmann's method corrected above.

The author's multiparametric system. We will now consider a set of ensembles consisting of N identical particles which is described by a set of asymmetrical distribution functions (8), which satisfy Liouville's equation and differ from one another solely in the initial data. If these ensembles of particles only interact elastically for a certain time with a simple thermostat, the system will be only slightly mixed and the distribution function will only be partially symmetrical. At the macroscopic level certain numbers of mean parameters (the densities, velocities and temperatures) will be preserved in the system. The distribution function will have the form

$$F_N = \prod_{s=1}^{l} \varphi_1(\mathbf{x}_s) \prod_{k=1}^{p} \varphi_2(\mathbf{x}_k) \ldots \prod_{t=1}^{\tau} \varphi_r(\mathbf{x}_t) \ldots \tag{13}$$

where $l + p + \ldots + \tau + \ldots M < N$.

The multiparticle, but identical, functions form a group

$$f_\tau = n_\tau \varphi_\tau(\mathbf{x}_1)$$

This dynamic system is characterized by the following set of particle functions. The number M at this level of consideration remains unknown. It is defined in the physical formulation of the problem itself. The particle functions satisfy a system of kinetic equations, obtained above using the corrected Boltzmann method. Bogolyubov's method for multicomponent systems has not yet been developed.

The particle functions $f_s(t, r)$, just like for multicomponent gaseous systems, will satisfy the following multiparametric system of kinetic equations

$$\frac{df_s}{dt} = \sum_{\tau=1}^{M} \iiint \left(f_\tau' f_s' - f_\tau f_s\right) q_{\tau s} b db d\varepsilon dv_\tau \tag{14}$$

This system is similar in its superficial form and mathematical content to the system of kinetic equations for gaseous mixtures of M flows. However, there is nothing in common between the systems as far as the physical basis is concerned. This system of equations was first obtained in our papers before 1980 when setting up the distribution function of groups of molecules [21] and is quite often encountered in publications by others.

The values of ρ_s° are known in problems on gaseous mixtures. We also know the number M. In the problems considered the values ρ_s° and M are unknown and must be defined in the physical formulation of the problem. Solving the multi-parametric system of kinetic equations using our method [15], we obtain the following multiparametric system of gas-dynamic equations

$$\begin{gathered}
\frac{\partial v_s^\alpha}{\partial t} + v_s^\beta \frac{\partial v_s^\alpha}{\partial r_\beta} + \frac{1}{\rho_s^\circ}\frac{\partial p_s}{\partial r_\alpha} = \frac{16}{3\rho_s}\sum_{\tau\neq s}^{M} \frac{\rho_\tau^\circ \rho_s^\circ \Omega_{\tau s}^{(11)}}{m_\tau + m_s}\left(v_\tau^\alpha - v_s^\alpha\right) \\
\frac{\partial T_s}{\partial t} + v_s^\alpha \frac{\partial T_s}{\partial r_\alpha} + \frac{2}{3}T_s \frac{\partial v_s^\alpha}{\partial r_\alpha} = \frac{16}{3}\sqrt{\frac{2\pi k T_s}{m_s}} \sum_{\tau\neq s}^{M} n_s \Omega_{\tau s}^{(11)} \left\{\frac{3k}{2m}\left(T_\alpha - T_s\right) + q_{\tau s}\left(v_\tau - v_s\right)^2\right\} \\
\frac{\partial \rho_s^\circ}{\partial t} + \frac{\partial \rho_s^\circ v_s^\alpha}{\partial r_\alpha} = 0; \quad q_{\tau s} = \frac{m_s T_s}{T_\tau - T_s}
\end{gathered} \tag{15}$$

The system of M equations of the generalized multiparametric gas-dynamic theory of matter of the twentieth century looks like this.

As can be seen, a system of M mutually connected partial differential equations has been obtained, where ρ_s° and the number M are, in general, unknown. They are defined in the physical formulation of the problem [20, 21].

7. We will use the multiparametric fluid-dynamic equations to describe the structure of a steady shock-wave. In this problem we will have two groups of molecules. The subscript one will relate to the group of molecules in front of the shock-wave and the subscript two will refer to the group of molecules behind the shock-wave.

If instead of the kinetic equation (14) or instead of the system of differential equations (15) we can use their conservation integrals, this will be the most important achievement.

From (15) we obtain the following equations for a one-dimensional steady wave: the equations of conservation of mass, angular momentum and total energy, and the equation for the thermal energy of the whole system

$$\begin{gathered}
\frac{d}{dx}\left(\rho_1 v_1 + \rho_2 v_2\right) = 0, \quad \frac{d}{dx}\left(\rho_1 v_1^2 + p_1 + \rho_2 v_2^2 + p_2\right) = 0 \\
\frac{d}{dx}\left(\frac{\rho_1 v_1^2}{2} + \frac{5}{3}p_1 v_1 + \frac{\rho_2 v_2^2}{2} + \frac{5}{3}p_2 v_2\right) = 0 \\
\frac{2}{3}\frac{d}{dx}\left(\left(\rho_1 v_1 T_1 + \rho_2 v_2 T_2\right) + p_1 k T_1 \frac{dv_1}{dx} + p_2 k T_2 \frac{dv_2}{dx}\right) = \frac{8}{3}\rho_1 \rho_2 \Omega_{12}^{(11)}\left(v_1 - v_2\right)^2 \Psi
\end{gathered} \tag{16}$$

where Ψ is a certain function of the flow parameters.

The system contains variables which relate to different groups of molecules $\rho_1, \rho_2, v_1, v_2, T_1, T_2$, where ρ is the density, v is the velocity and T is the temperature.

In this theory the flow is formed from groups of molecules. The first group exists in front of the compression jump, its density is specified, while behind the jump $\rho_1(x) = 0$, and for the second group $\rho_2(x) = 0$ in front of the jump. These quantities are the boundary conditions for the problem in question on the structure of intense shock-waves.

In shock problems, as we know, the density and other parameters of the flow are specified at the shock front, and all the parameters behind the shock are calculated from the Rankin–Hugoniot relations.

In the problem considered here on the structure of high-intensity shock-waves, the boundary conditions

enable us to determine the change in the gas density using Eqs (16). The quantities v_1, v_2, T_1 and T_2 are taken to be constant. The ratios v_2/v_1 and T_2/T_1 can be found from the condition for the first three equations of (16) to be compatible from the formulae

$$\frac{v_2}{v_1}=\frac{M^2+3}{4M^2},\quad \frac{T_2}{T_1}=\frac{(M^2+3)(5M^2-1)}{16M^2}$$

We will now introduce the mean parameters of the flow, which characterize the system as a whole

$$\rho(x)=\rho_1(x)+\rho_2(x),\quad v(x)=\frac{\rho_1 v_1+\rho_2 v_2}{\rho_1+\rho_2},\quad T(x)=\frac{\rho_1(x)T_1+\rho_2(x)T_2}{\rho_1+\rho_2}$$

To determine $\rho_1(x)$ and $\rho_2(x)$ we will have

$$v_1\rho_1(x)+v_2\rho_2(x)=c_1,\quad \frac{d}{dx}(\rho_1 v_1 kT_1+\rho_2 v_2 kT_2)=\frac{16}{9}\rho_1\rho_2\Omega_{12}^{(11)}(v_1-v_2)^2\Psi \tag{17}$$

Assuming

$$y_1=\frac{\rho_1(x)}{\rho_{-\infty}},\quad y_2=\frac{\rho_2(x)}{\rho_{-\infty}},$$

we obtain

$$v_1y_1+v_2y_2=v_1,\quad -\frac{dy_1}{y_1(1-y_1)}=Adx \tag{18}$$

where A and Ψ are complex functions of the flow parameters, independent of the coordinates. Solving system (18) we obtain

$$\rho_1(x)=-\frac{\rho_{-\infty}}{1+e^{-Ax}},\quad \rho_2(x)=\frac{\rho_{-\infty}}{1+e^{+Ax}}\frac{v_1}{v_2} \tag{19}$$

Following Prandtl, we define the shock-wave thickness

$$\delta=\frac{\rho_{+\infty}-\rho_{-\infty}}{(d\rho/dx)_{\max}}$$

We will denote the ratio of the mean free path λ to the shock-wave thickness δ by $f(M)$. We can only determine this quantity $\lambda/\delta=f(M)$ in specially set-up experiments.

Figure 1 (see also [22–24]) shows a graph of λ/δ against the Mach number.

Proceeding as in Schmid's paper [25] and using his notation, we show in Fig. 2 the change in the normalized density $\rho_N(x)$ over the width of the jump, where ρ_2 and ρ_1 are the values of the density at $\pm\infty$

$$\rho_N(x)=\frac{\rho(x)-\rho_1}{\rho_2-\rho_1} \tag{20}$$

When solving the problem of the shock-wave structure we ignored the effect of the viscosity and the thermal conductivity. It is primarily necessary to take into proper account the effect of these terms in the equations themselves (see [15]).

8. We will use the multiparametric kinetic theory to describe turbulent flows in a plane channel. We will first write Eqs (15) for the steady one-dimensional flow of an incompressible viscous fluid in a channel $y=\pm h$ between two parallel planes

$$\rho\left(\frac{\partial U_s U_s}{\partial x}+\frac{\partial U_s v_s}{\partial y}\right)+\frac{\partial p_s}{\partial x}=\sum_{\tau\neq s}^{M}Q_{\tau s}(U_\tau-U_s)+\mu\bar{V}^2U_s$$

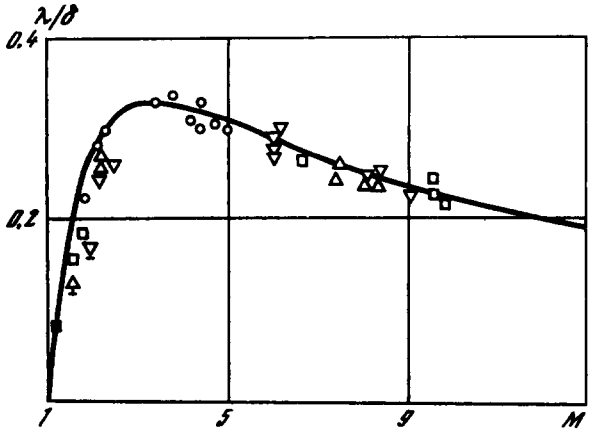

Fig. 1.

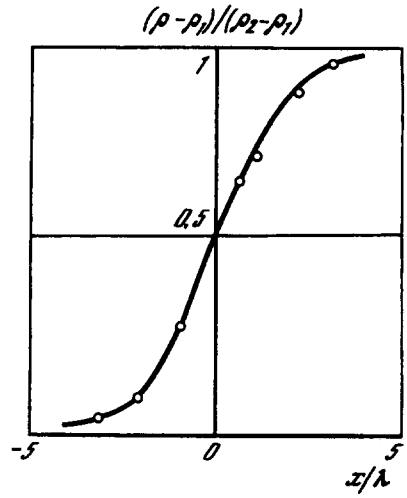

Fig. 2.

$$\rho\left(\frac{\partial U_s v_s}{\partial x}+\frac{\partial v_s v_s}{\partial y}\right)+\frac{\partial p_s}{\partial y}=\sum_{\tau\neq s}^{M} Q_{\tau s}(v_\tau - v_s)+\mu\overline{V}^2 v_s \tag{21}$$

$$\frac{\partial U_s}{\partial x}+\frac{\partial v_s}{\partial y}=0;\quad Q_{\tau s}=\frac{8n}{3}\Omega_{\tau s}^{(11)}$$

Here $\Omega_{\tau s}$ are integrals, given and partially tabulated in [2, 3]. For one-component gases the value of $\Omega_{\tau s}$ are constant quantities. Hence, we will use the standard arithmetic-mean law of averaging the parameters of the flow.

In this paper the viscous terms in Eqs (21), as in [21], are written in classical form, ignoring, for the present, any large correction (see [15] and other papers by the author on this issue).

As a result of a comparison of theory and experimental data it became clear that in turbulent flow, friction at the channel walls is greater than the theory predicts. Hence, the large correction to the viscous terms mentioned above, which increase the friction forces and their role [15], must henceforth be taken into account in the theory.

We will introduce averaging of the flow parameters over their set of possible values

$$\overline{U}=\frac{1}{M}\sum_{\tau=1}^{M} U_\tau,\quad \overline{v}=\frac{1}{M}\sum_{s=1}^{M} v_s,\quad \overline{p}=\frac{1}{M}\sum_{\tau=1}^{M} p_\tau \tag{22}$$

We can then naturally introduce a set of deviations from the mean parameters of the flow (denoted by an asterisk) rather than the set of pulsation, as it was from

$$U_s=\overline{U}+U_s^*,\quad v_s=\overline{v}+v_s^*,\quad p_s=\overline{p}+p_s^* \tag{23}$$

It can be seen that the sums in Eqs (21) take the form

$$\sum_{\tau\neq s}^{M}\{U\}_{\tau s}=-MU_s^*,\quad \sum_{\tau\neq s}^{M}\{v\}_{\tau s}=-Mv_s^* \tag{24}$$

Substituting (24) into (21) and carrying out the averaging, we obtain the following system of equations for the mean parameters of the flow

$$\rho\left(\frac{\partial\overline{U}\,\overline{U}}{\partial x}+\frac{\partial\overline{U}\,\overline{v}}{\partial y}\right)+\frac{\partial\overline{p}}{\partial x}=\mu\nabla\overline{U}-\rho\left(\frac{\partial\overline{U_s^*U_s^*}}{\partial x}+\frac{\partial\overline{U_s^*v_s^*}}{\partial y}\right) \tag{25}$$

$$\rho\left(\frac{\partial\bar{v}\,\bar{U}}{\partial x}+\frac{\partial\bar{v}\,\bar{v}}{\partial y}\right)+\frac{\partial\bar{p}}{\partial y}=\mu\nabla\bar{v}-\rho\left(\frac{\partial\overline{U_s^*U_s^*}}{\partial x}+\frac{\partial\overline{v_s^*v_s^*}}{\partial y}\right)$$

$$\partial\bar{U}/\partial x+\partial\bar{v}/\partial y=0$$

This system can be simplified considerably for the one-dimensional problem considered

$$\frac{d^2\bar{U}}{dy^2}=\frac{1}{\mu}\frac{\partial\bar{p}}{\partial x}+\frac{\rho}{\mu}\left(\frac{\partial\overline{U_s^*U_s^*}}{\partial x}+\frac{\partial\overline{U_s^*v_s^*}}{\partial y}\right) \tag{26}$$

The system of equations for the deviations of the mean parameters of the flow has a more complex form than (25). However, for simplifications of the one-dimensional problem it can be reduced to the form

$$\frac{d^2U_s^*}{dy^2}-k^2U_s^*=\frac{1}{\mu}\frac{\partial p^*}{\partial x},\quad \frac{d^2v_s^*}{dy^2}-k^2v_s^*=\frac{1}{\mu}\frac{\partial p^*}{\partial y},\quad \frac{dU_s^*}{dx}+\frac{dv_s^*}{dy}=0 \tag{27}$$

where $k^2 = M/\mu$ is found by comparing a series of theoretical calculations with experimental data.

The system of equations (27) turned out to be still quite complex. Hence, the values of the deviations of the flow parameters are determined approximately. Using these one can calculate the mean turbulent velocity of the flow in a channel from Eq. (26).

Hence, in this paper we have considered a number of dynamical and statistical systems in general form and we have shown that one can construct an extremely non-uniform kinetic theory which should now operate with a set of M single-particle distribution functions which satisfy the multiparametric system of M kinetic equations (14). This system of kinetic equations has been reduced, by a method similar to that described earlier [15], to a multiparametric system of M partial differential equations (15).

The new theory has been used to calculate the shock-wave structure.

A comparison of the theory with experiment has exceeded all expectations. As can be seen, the shock-wave thickness agrees with experiment over a wide range of Mach numbers with remarkable accuracy (Fig. 1). This is also confirmed by a comparison of the theory with experiment for the density distribution across a channel (Fig. 2).

Calculations of the distribution of the turbulent velocities in a plane channel agree well with experimental data for the axial section of a circular tube (see Fig. 3, where the curve with the dark points represents Nikuradze's experiment [19], the curve with the crosses represents the calculation of the turbulent flow, while the curve with the small circles represents laminar flow).

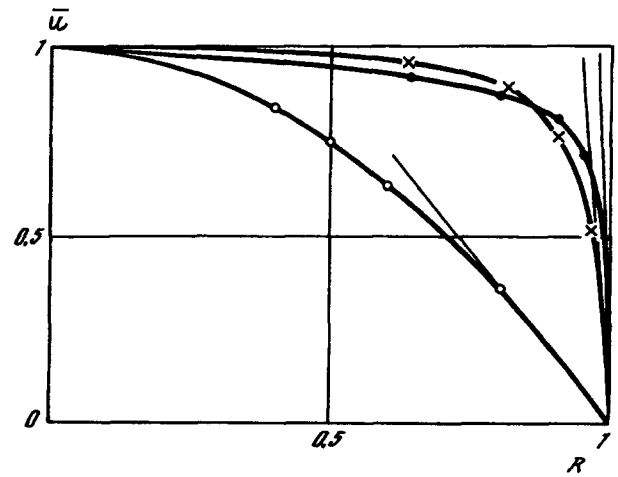

Fig. 3.

Viscous friction, according to experimental data, turns out to be somewhat greater than in theory. Hence, corrections to the viscous terms in Eqs (15) must be made afterwards.

At the same time, we can only be amazed that the theory simultaneously correctly describes the structure of the turbulent core at the channel centre and also quite correctly the laminar sublayer on the channel wall.

In Fig. 3 we show the changes in the relative value of the turbulent velocity of the flow with respect to its value at the tube centre. To determine the true values of the parameters of turbulent flow one must start from Eqs (21), into which one must introduce bulky expressions for viscous friction, calculated in [15] or other papers we have written on the same issue.

To determine the zone in which a transition from laminar flow to turbulent flow occurs one must start from the multiparametric system of gas-dynamic equations (15), adding to them viscous terms calculated in [15] and other papers.

The new theory of turbulence undoubtedly needs to be checked further on other more complex aerodynamic processes and used to solve important basic problems in science and technology.

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